

Proving Trig Identities Worksheet 2

Use the Reciprocal and Quotient identities to prove the following.

$$\begin{aligned}
 1.) \sin x \sec x &= \tan x \\
 &= \sin x \left(\frac{1}{\cos x} \right) = \frac{\sin x}{\cos x} \\
 &= \tan x \quad \text{☺}
 \end{aligned}$$

$$\begin{aligned}
 2.) \tan x &= \frac{\sec x}{\csc x} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} \\
 &= \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \tan x \quad \text{☺}
 \end{aligned}$$

$$\begin{aligned}
 3.) \sin x \csc x &= 1 \\
 \sin x \left(\frac{1}{\sin x} \right) &= \frac{\sin x}{\sin x} = 1 \quad \text{☺}
 \end{aligned}$$

$$\begin{aligned}
 4.) \cot x &= \frac{\csc x}{\sec x} = \frac{\frac{1}{\sin x}}{\frac{1}{\cos x}} \\
 &= \frac{1}{\sin x} \cdot \frac{\cos x}{1} = \frac{\cos x}{\sin x} = \cot x \quad \text{☺}
 \end{aligned}$$

$$5.) \frac{\tan^2 x}{\sec^2 x} + \frac{\cot^2 x}{\csc^2 x} = 1$$

$$\begin{aligned}
 &= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} + \frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}} \\
 &= \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x + \frac{\cos^2 x}{\sin^2 x} \cdot \sin^2 x
 \end{aligned}$$

$$= \sin^2 x + \cos^2 x = 1 \quad \text{☺}$$

$$6.) \cos^2 x = \frac{\cos x}{\sec x}$$

$$\begin{aligned}
 &= \frac{\cos x}{\frac{1}{\cos x}} \\
 &= \cos x \cdot \cos x \\
 &= \cos^2 x \quad \text{☺}
 \end{aligned}$$

$$7.) \frac{1}{\sin x} - \frac{1}{\csc x} = \csc x - \sin x$$

$(\csc x)$
 $(\sin x)$

$(\csc x)$
 $(\sin x)$

$$\frac{\csc x}{\sin x \csc x} - \frac{\sin x}{\sin x \csc x} = \frac{\csc x - \sin x}{\sin x \csc x}$$

$$= \frac{\csc x - \sin x}{\sin x \left(\frac{1}{\sin x}\right)} = \csc x - \sin x$$

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$$9.) \sin x \csc x \sec x = \frac{1}{\cos x}$$

$$= \sin x \left(\frac{1}{\sin x}\right) \left(\frac{1}{\cos x}\right) = \frac{1}{\cos x}$$

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$$8.) \csc x = \frac{\sec x}{\tan x}$$

$$= \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1}{\sin x} = \csc x$$

"

$$10.) \cos x = \frac{\cos x \tan x}{\sin x \sec x}$$

$$= \frac{\cos x \left(\frac{\sin x}{\cos x}\right)}{\sin x \left(\frac{1}{\cos x}\right)}$$

$$= \frac{\sin x}{\sin x} \cdot \frac{\cos x}{1} = \cos x$$

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